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Relativistic non-covariance in interacting spin-1 field theories

K Babu Joseph and M Sabir

Department of Physics, Cochin University, Cochin-682022, India

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Abstract. Acausal interactions of a massive spin-1 field such as symmetric tensor and quadrupole moment couplings are not covariant at the basic c-number level. Curiously enough, certain causal derivative couplings are also non-covariant.

1. Introduction

Field theory of high-spin particle interactions is plagued by a number of difficulties. One of the most serious of these is causality violation at the classical level as illustrated by the work of Velo and Zwanziger (1969a, b). Jenkins (1973, 1974), who first observed that the quantized version of a pathological theory is not covariant, conjectured that acausal theories are also non-covariant at the classical level. The type of non-covariance Jenkins has noted stems from the impossibility of simultaneously determining the dependent components in all frames of reference. Motivated by this, we have studied the covariance problem in certain particularly simple pathological theories within the Poincaré group framework. We find that when such an interaction is present the structure relations of the Poincaré group are not satisfied and the theories do not possess relativistic covariance. Two types of interactions of a massive spin-1 particle with an external field exhibiting this abnormal behaviour are presented as examples. A brief discussion is also given of the covariance question in derivatively-coupled systems of the massive vector and Dirac fields.

2. Non-covariance

In order that a field theory be relativistically covariant the ten generators $P^{\mu}, J^{\mu\nu}$ constructed in terms of the fundamental field variables must obey the structure relations characterizing the geometric nature of the Poincaré group. Since the present discussion is almost wholly confined to the classical level, we make use of the Poisson bracket realization of the Poincaré group:

$$
\begin{aligned}
&[P_{\mu}, P_{\nu}] = 0 \\
&[J_{\mu\nu}, P_{\rho}] = g_{\mu\rho}P_{\nu} - g_{\nu\rho}P_{\mu} \qquad\qquad (1)\\
&[J_{\mu\nu}, J_{\rho\sigma}] = g_{\nu\sigma}J_{\mu\rho} - g_{\mu\sigma}J_{\nu\rho} - g_{\nu\rho}J_{\mu\sigma} + g_{\mu\sigma}J_{\nu\rho}.
\end{aligned}
$$

Only two of these relations are the necessary conditions for Lorentz covariance in its proper sense; the others are related to the three-dimensional aspects and may be treated as trivial conditions. The significant relations are

$$[J^{0k}, P^0] = -P^k \qquad [J^{0k}, J^{0l}] = -J^{kl}. \tag{2}$$

These lead to the well known Dirac–Schwinger covariance condition (Dirac 1962, Schwinger 1962). In the theories we present $[T^{00}(x), T^{00}(x')]$ contains in addition to $-[T^{0k}(x)+T^{0k}(x')]\,\partial_k\delta(\boldsymbol{x}-\boldsymbol{x}')$ a term of the form $[f^{0k}(x)+f^{0k}(x')]\,\partial_k\delta(\boldsymbol{x}-\boldsymbol{x}')$. Since the term is not in the form of a three-divergence it is impossible to satisfy the structure relations (2).

3. Examples

The first example we consider is the interaction of a massive vector field $\phi_\mu(x)$ with an external symmetric tensor field $W^{\mu\nu}$. The Lagrangian is

$$\mathscr{L} = -\tfrac{1}{2}G^{\mu\nu}(\partial_\mu\phi_\nu - \partial_\nu\phi_\mu) + \tfrac{1}{4}G^{\mu\nu}G_{\mu\nu} - \tfrac{1}{2}m^2\phi^\mu\phi_\mu + \tfrac{1}{2}\lambda\phi^\mu W_{\mu\nu}\phi^\nu. \tag{3}$$

The abnormalities inherent in this theory have already been discussed by Velo and Zwanziger (1969b) and by Jenkins (1974). The energy and momentum densities of this field system are

$$T^{00} = \tfrac{1}{2}[(G^{0k})^2 + \tfrac{1}{2}(\partial_k\phi_l - \partial_l\phi_k)^2 + m^2\phi_k^2 + (m^2+\lambda W^{00})^{-1}(\partial_k G^{0k})^2] \tag{4}$$

$$T^{0k} = (\partial^k\phi_l)G^{0l}. \tag{5}$$

Here, it is assumed, for simplicity, that only the W^{00} component of the symmetric tensor field is non-vanishing. It may easily be verified that this assumption does not invalidate any of the contentions of the present paper. Making use of the basic Poisson bracket relations

$$\begin{aligned} &[\phi_k(x), G^{0l}(x')] = -\delta_k^l\delta(\boldsymbol{x}-\boldsymbol{x}') \\ &[\phi_k(x), \phi_l(x')] = [G_{0k}(x), G_{0k}(x')] = 0 \end{aligned} \tag{6}$$

the Poisson bracket $[T^{00}(x), T^{00}(x')]$ is evaluated:

$$\begin{aligned} [T^{00}(x), T^{00}(x')] &= -\left(\partial^k\phi_l(x)G^{0l}(x) + \frac{\lambda W^{00}}{m^2+\lambda W^{00}}\phi^k\partial_l G^{0l} + \partial'_k\phi_l(x')G^{0l}(x')\right. \\ &\quad \left. + \frac{\lambda W^{00}}{m^2+\lambda W^{00}}\phi^k(x')\partial'_l G^{0l}(x')\right)\partial_k\delta(\boldsymbol{x}-\boldsymbol{x}'). \end{aligned} \tag{7}$$

Within the brackets on the right-hand side of the equation, the term $f^{0k} = [\lambda W^{00}/(m^2+\lambda W^{00})]\phi^k\,\partial_l G^{0l}$ appears in addition to T^{0k} and this implies, as we have already argued, that the theory is non-covariant.

The second example is provided by the quadrupole coupling of a massive vector field to an external electromagnetic field with the interaction Lagrangian

$$\mathscr{L}_{\mathrm{I}} = g\phi_\lambda Q^\lambda_{\mu\nu}\,\partial^\mu\phi^\nu \tag{8}$$

where $Q^\lambda_{\mu\nu} = \partial^\lambda F_{\mu\nu}$ and $F_{\mu\nu}$ is the field tensor of the electromagnetic field.

In the simple case where the external field is an electrostatic field we have

$$T^{00} = -\partial_k G^{0k}\phi_0 + \tfrac{1}{2}(G^{0k})^2 + \tfrac{1}{4}(\partial_k\phi_l - \partial_l\phi_k)^2 + \tfrac{1}{2}m^2(\phi_k^2 - \phi_0^2)$$
$$T^{0k} = (G^{0l} - g\phi^\lambda Q_\lambda^{0l})\,\partial^k\phi_l \tag{9}$$

where

$$\phi^0 = \frac{-[\partial_k G^{0k} + g\,\partial_i(\phi^k\,\partial_k E^i)]}{m^2 + g\partial_i E^i}.$$

In the energy-density Poisson bracket there is an additional term given by

$$f^{0k} = g[G^{0l}\phi_0\,\partial_l E^k + \phi_0^2\,\partial^k\,\partial_i E^i + \phi^k\,\partial_i(\phi_l\,\partial^l E^i) + \phi^l(\partial^k\phi_i)\,\partial_l E^i]. \tag{10}$$

Since this term cannot be written as a three-divergence it follows that the theory is non-covariant.

4. Coupling with a Dirac field

In the case of mutual interaction between a massive vector field and a Dirac field it is known that only the derivative couplings of scalar, pseudoscalar and pseudovector type lead to acausal propagation (Shamaly and Capri 1972). While we have proved that these theories are not covariant, we have, to our surprise found that the other derivative couplings (vector and tensor), wherein no pathologies were previously reported, also do not possess Lorentz covariance. The energy-density Poisson brackets for the causal couplings

$$\mathcal{L}_{I_1} = f\bar\psi\gamma^\mu\psi\,\partial_\mu(\phi^\nu\phi_\nu) \qquad \text{and} \qquad \mathcal{L}_{I_2} = g\bar\psi\sigma^{\mu\nu}\psi\,\partial_\mu\phi_\nu$$

contain the extra terms

$$f^{0k}_{\mathcal{L}_{I_1}} = f(\bar\psi\gamma^l\psi)\,\partial_l(\partial_j G^{0j})\phi^k/m^2 \tag{11}$$
$$f^{0k}_{\mathcal{L}_{I_2}} = -g(\bar\psi\gamma_0\psi G^{0k} + \bar\psi\sigma^{kl}\psi\phi_l) \tag{12}$$

which are not three-dimensional divergences.

5. Conclusion

As a concluding remark, it is noted that the inference regarding the non-covariance of the above theories holds good in the quantized versions as well, if the basic Poisson bracket relations of the field are replaced by commutation (anti-commutation) relations. Though Jenkins (1973) has observed that the quantized theories are non-covariant, it is in the sense that no covariant S matrix can be defined in the interaction picture. But the present investigation demonstrates the fact that such theories are not covariant in the Heisenberg picture itself.

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